Titolo del corso: **Topics in stochastic Analysis: Jump processes and Malliavin Calculus**

Docenti: **Elena Bandini** (Università di Bologna), **Arturo Kohatsu-Higa** (Ritsumeikan University)

Membri del collegio proponenti: Stefano Pagliarani, Andrea Pascucci

Ore frontali di lezione: **20 (10+10)**

Periodo di lezione: **marzo/giugno 2026**

Settore/i disciplinare del corso: **MAT/06**

Tipologia di corso: **Base**

Modalità di verifica dell'apprendimento: **Esame scritto**

Abstract del corso:

Module 1 (Elena Bandini): The objective of the course is to introduce stochastic calculus for jump processes. Starting from the main definitions and results of discontinuous stochastic processes theory, we introduce the stochastic integral with respect to finite variation processes and we investigate its main properties. Moreover, we study stochastic differential equations driven by jump processes, showing an existence and uniqueness result. Finally, we present the Doléans-Dade exponential, and we study the martingale property of the stochastic integral.

Module 2 (Arturo Kohatsu-Higa): Malliavin calculus, also known as the stochastic calculus of variations, provides tools for analyzing the smoothness of functionals of stochastic processes. It is particularly useful in understanding the regularity properties of solutions to stochastic differential equations (SDEs), including those arising in mathematical finance. For example, Malliavin calculus can be used to study the smoothness of the density, which is crucial for practical applications like numerical simulations and approximations of prices and sensitivities.

Programma del corso:

Module I:

- Introduction to pure jump processes

- The Poisson point process and Watanabe theorem

- Introduction to stochastic integrals with respect to finite variation processes

- Stochastic integral with respect to finite variation processes

- Introduction to stochastic differential equations driven by pure jump processes

Module II:

- Stochastic derivative

- Chaine rule

- Clark-Ocone formula
- Applications to mathematical finance